JOMTA
Journal of Mathematics: Theory and Applications
Vol. 5, No. 1, 2023, P-ISSN 2685-9653 e-ISSN 2722-2705
DOI: 10.31605/jomta.v5i1.1762

Parameter Estimation of The Blumberg Model Using Simulated Annealing Algorithm: Case Study of Broiler Body Weight

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Abstract. *The Blumberg model is one of the logistic models. The advantage of the Blumberg model is the flexibility of the inflection point. The Blumberg model is believed to be suitable for modeling the growth of living organs. In this article, we estimate the parameters of the Blumberg model using simulated annealing algorithm. The simulated annealing algorithm is a heuristic optimization method based on the metal annealing process. The data used is Broiler daily weight data. The model obtained fits the daily weight data of Broiler. Our results show that the closer the cooling schedule factor to 1, the smaller the error. In addition, we must carefully select the initial temperature. The selection of the initial temperature that is not suitable drives the error to enlarge.*

Keywords*:Blumberg model, simulated annealing algorithm, parameter estimation , Broiler; heuristic method*

I. INTRODUCTION

Mathematical models are very powerful tools for describing real-world problems. Broadly speaking, mathematical modeling is carried out through four steps, namely problem identification, construction of mathematical models, determining model solutions, and interpreting model solutions [1]. In general, mathematical models consist of independent variables, dependent variables, and parameters. In the field of mathematical biology, there are single-species models and interacting species models. Usually, the single-species model only consists of one dependent variable. Meanwhile, the interacting species model consists of several dependent variables.

One of the most vital things in mathematical modeling is the determination of parameter values. Some researchers use parameter values that are theoretical in nature so that the model solution is only illustrative. On the other hand, there are also researchers who use parameter values based on estimation results using observational data so that the model solution is more applicable to be used in solving real-world problems [2]–[4].

When observational data is available, it can be used to estimate the value of the model parameters so that the model solution approximates the observation data in the field. A good model is a model whose solution is in accordance with the observation data [1]. When estimating the parameters, the thing that will be faced is the optimization problem, which is to minimize the error between the model solution and the observation data. There are several methods that can be used to solve optimization problems to minimize errors, namely methods that involve derivatives such as the Newton-Raphson method and steepest descent [5] or methods that do not involve derivatives such as genetic algorithm [6], particle swarm optimization, Nelder-Mead [7], and simulated annealing (SA) [8], [9].

The basis of the SA algorithm is in thermodynamic problems, where one studies the thermal energy of a system. The cooling process after heating the metal motivates the algorithm [9]. The SA algorithm is one of the heuristic methods that can be used to find the global optimum solution to the optimization problem [8].

The estimation of Broiler growth model parameters has been discussed in [1]. However, the model used is the Verhulst logistic growth model. In 1968, Blumberg [10] developed a logistic growth model. The model proposed is suitable for the growth of organs [11]. In this article, we estimate the parameters of the Blumberg model using a SA algorithm. The data used is Broiler daily weight data.

II. THEORETICAL BASIS

2.1 Blumberg Model

In 1968, Blumberg proposed an equation that is an extension of the Verhulst equation to model population dynamics or organ size evolution. The equation has two extra parameters, e.g., shape parameters, which can drive the equation equivalent to Gompertz equation [11]. Blumberg model is more flexible at the point of inflection. The model is believed to be a suitable model for the case of organ growth in living things. Blumberg model is expressed as

$$\frac{dW}{dt} = rW^{\beta}\left(1 - \frac{W}{K}\right)^{\gamma}, \qquad (1)$$

where W is the population size, t is time, β and γ are shape parameters, r is Malthusian parameter, and K is carrying capacity.

2.2 Simulated Annealing Algorithm

SA algorithm is one of the heuristic optimization algorithms. As previously mentioned, this algorithm is inspired by the metal annealing process. SA is widely utilized to solve global optimization problems. SA algorithm flowchart is shown in Figure 1. The figure is taken from [12]. In the SA algorithm, there are two very crucial features, namely initial temperature and cooling schedule. The Boltzmann distribution plays an important role in SA. They associate the probability of a state x, its energy $f(x)$, and temperature $T > 0$ through:

$$\pi_T(x) = \frac{1}{Z_T}\exp\left(-\frac{f(x)}{T}\right).$$

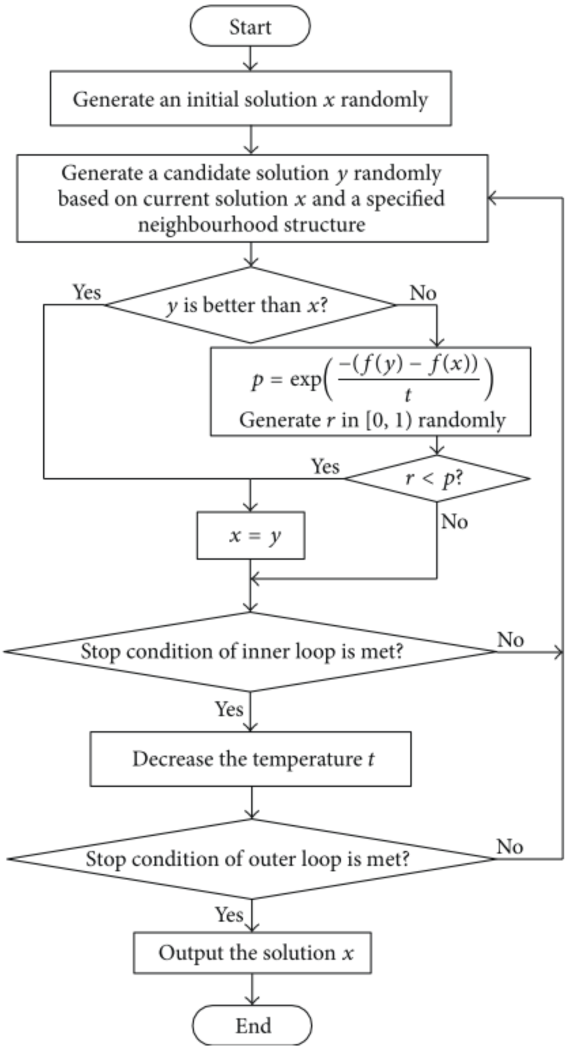

Figure 1. SA algorithm flowchart

III. METHOD

This research was conducted in accordance with the following research stages:

- ✓ Collecting data: the data used is taken from https://en.aviagen.com/tech-center;
- ✓ Specify the model: in this paper, we use Blumberg model given by (1). Here, W represent the weight of Broiler, while r, β, K, and γ are parameters that will be estimated;
- ✓ Determine the objective function;
- ✓ Choose an optimization method: In this study, we chose the SA algorithm;
- ✓ Create a SA program using MATLAB based on the SA algorithm flowchart;
- ✓ Specifies the initial value of the parameters;
- ✓ Running the SA program using MATLAB.

IV. RESULTS AND DISCUSSIONS

We estimate r, β, K, and γ in equation (1) by trying several initial temperature t_0 and α. On the other hand, we fixed the final temperature, i.e., $t_{end} = 10^{-10}$. We use a geometric cooling schedule, i.e. $t = \alpha^n t_0$ for $n = 1, 2, n_{end}$. After inputting the parameter values into the input part of the SA program, we run the SA program. We run the SA program seven times by changing the input for t_0 and α. The results are shown in Table 1.

Table 1. Parameter estimation results

Exp.	Input			Output			
	t_0	α	r	K	β	γ	**MAPE**
1	10^8	0.75	0.921	3018.341	0.821	0.621	0.759
2	10^8	0.80	0.927	3030.910	0.827	0.627	0.756
3	10^8	0.85	0.950	3076.399	0.850	0.650	0.745
4	10^8	0.90	0.958	3091.858	0.858	0.658	0.740
5	10^8	0.95	0.981	3138.496	0.881	0.681	0.725
6	10^{10}	0.95	0.983	3141.124	0.883	0.683	0.720
7	10^{11}	0.95	1.029	3058.785	0.829	0.929	0.746

Our experimental results as given in Table 1 show that the closer the α to 1, the smaller the error. Note that the determination of t_0 must be done carefully. Choosing a larger t_0 does not guarantee to reduce the error.

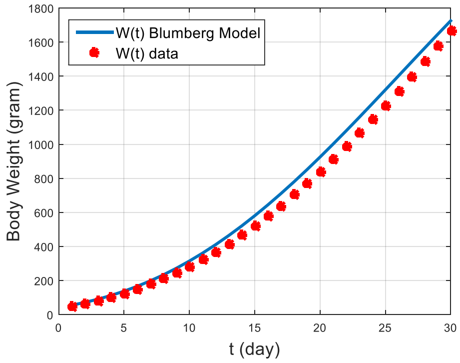

Figure 2. The daily data and model solution curves for the sixth experiment

Based on the MAPE (mean absolute percentage error) values shown in Table 1, the sixth experiment gives the best estimation results. The daily data curve and model solution curve for the sixth experiment are presented in Figure 2. In Figure 2, we cannot see the inflection point yet. From the slope of the curves shown in Figure 2, it is clear that the Broiler growth rate after the 10th day is greater than the growth rate in the first ten days. Further, it can be seen that the model solution curve fits the daily data curve fairly well. Therefore, the Blumberg model that fits the Broiler weight data used is

$$\frac{dW}{dt} = 0.983 \times W^{0.883}\left(1 - \frac{W}{3141.124}\right)^{0.683}.$$

Next, we estimate the weight of the Broiler using the parameter values obtained from the sixth experiment. The results are shown in Figure 3. Based on the slope of the curve given in Figure 3, it is clear that the Broiler growth rate started to decrease between the 30th and 40th day. In contrast to Figure 2, in Figure 3 we can already observe the point of infection which occurred between the 30th and 40th day. These results tell us that the need to maximize broiler growth is in the first 30 days. Furthermore, it can be seen that the weight of the Broiler on the 60th day reaches 3 kilograms.

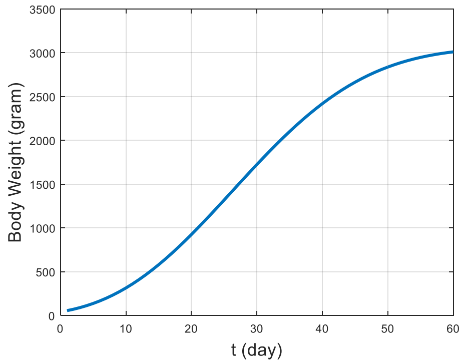

Figure 3. Broiler daily weight estimation

V. CONCLUSIONS

We have discussed the estimation of the parameters of the Blumberg model using SA algorithm. The model obtained is quite compatible with the daily weight data of Broiler. Our experimental results show that the closer the cooling schedule factor value to 1, the smaller the error. Note that the initial temperature determination must be done carefully. Choosing an unsuitable initial temperature can increase the error.

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