Design 3D Wallpaper Motifs from Sierpinski Carpet Fractals Using Mathematica Applications

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Abstract

Sierpinski carpet is a unique fractal geometric form. Sierpinski carpet is built through recursive equations with patterns obtained from repeated iterations. This article uses Wolfram Mathematica® software to explore the application of 2D Sierpinski Carpet fractals to produce unique and aesthetically appealing 3D wallpaper motifs. This article shows an attempt to build/design a 3D Sierpinski carpet motif pattern using the concept of recursive iteration, fractal dimension parameterization, and visualization techniques. The results obtained from the Wolfram Mathematica® algorithm and programming created are fractal geometric forms that can be varied aesthetically into complex 3D wallpaper motifs. The output of the program created can be developed for various 3D batik motif designs.

Keywords: Fractal 3D, Sierpinski Carpet Square, Wolfram Mathematica®

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1. Introduction

Wallpaper was first discovered in China 200 years before Christ. Made from paper decorated by hand using ink with images of birds or flowers and attached using adhesive made from rice. Even though it looks very simple, wallpaper made in China symbolizes religious symbols.

The development of mathematics and computer science is a supporting factor in developing variations in wallpaper motifs. Symmetrical patterns and repeating shapes in motifs can be depicted fractally. The word fractal was first coined by Mandelbrot in 1975, when his paper entitled "A Theory of Fractal Sets" was published [1].

According to Sampurno and Faryui, in the book Fractal Analysis Methods they say "Fractals are a collection of geometric patterns both found in nature and in the form of visualizations of mathematical models where the pattern is repeated many times on an increasingly smaller scale." Fractal motifs are created using mathematical calculations which are visualized with a computerized system so that they become certain images [2]. Geometry comes from the Greek geo, which means earth, and metria, which means measurement. Geometry can be said to be the science of studying the shapes and sizes of the earth [3]. Beside that, geometry combines the abstract presentation of visual and spatial experiences, for example planes, patterns, measurements and mapping [4].

2. Literature Review and Methodology

2.1 Fractal Geometry

The study of fractals has been studied long before the word fractal was used. The word fractal was first coined by Mandelbrot in 1975 (see [1]). Meanwhile, the root of the word fractal comes from the Latin word "frangere" which means split into irregular fragments. Fractals are simple geometries that can be broken down into several parts that have a shape like the previous shape with a smaller size (see [1], [5], [6]). Fractals are a collection of geometric patterns both found in nature and in the form of visualizations

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of mathematical models where the pattern is repeated many times on an increasingly smaller scale. The resulting shape can be divided into several parts, each of which has a similar shape to each other (self-similarity) [7], [8]. Fractals have unlimited detail, or in the sense that the arrangement of the multiples is not bound by any orientation rules, in fact they tend to twist and turn in varying sizes from small to large.

Some fractals, if broken down and taken out of several small parts, will look similar to the original fractal. Fractals are said to have infinite detail and at different levels of magnification [9]. The various types of fractals were originally studied as mathematical objects. Fractals are geometric patterns that repeat starting from a small scale to produce irregular shapes that cannot be described using classical geometry [6].

Fractals are often found in nature, such as in the patterns found on tree leaves and twigs, in broccoli vegetables, in clusters of white clouds, in the ripples of waves, in details that can be seen in snowflakes, and many more around us. The following are examples of fractals found in nature:



Figure 1. Broccoli [10] and Barnsley's fern [11]

Fractals also have characteristics in terms of a mathematical parameter, namely fractal dimension. Fractal sets have five characters, namely [6]:

- 1. It exhibits a smooth structure, no matter how much it is magnified.
- 2. It is excessively irregular if described in ordinary geometric language.
- 3. It possesses self-similarity, either approximately or statistically.
- 4. The fractal dimension is typically greater than its topological dimension.
- 5. Generally, it can be defined in a simple manner, possibly recursively.

Fractal geometry provides a description and mathematical model of complex events in nature that is different from the known Euclidian geometry [6]. Objects in Euclidian geometry are described using formulas, while in fractal geometry they are described using an iterative algorithm [9]. Fractals have properties.

- 1. Self-Similarity:
 - Self-similarity indicates that a fractal consists of parts that are similar to one another.
- 2. Self-Affinity:

Self-affinity describes the arrangement of fractals in interconnected parts.

3. Self-Inverse:

Self-inverse means that a portion of the fractal can be an inverted arrangement of another portion.

4. Self-Squaring:

Self-squaring can be interpreted as a part of the fractal being an increased complexity of the previous part.

The following is an example of a fractal [12], [13]:



Figure 2. Fractal trees and Julia fractals

2.2 Sierpinski Carpet

The Sierpinski carpet is not much different from the Sierpinski Triangle. The construction of the Sierpinski carpet begins with a full square plane (see [14], [15]).



Figure 3. Five diagrams of Sierpinski carpet

This set can be expressed as a combination of eight sub-sets that are congruent to the original set and have a scale with factors $\frac{1}{3}$. Thus, these sub-sets have a value of k=8 and a scale factor of $s = \frac{1}{3}$ and it can be seen that the square pattern in this set will continue to repeat itself with increasingly smaller scale factor values. Shidu illustrates the mathematical equation for generating Sierpinski carpets, namely [15]:

$$W_{i}\begin{bmatrix} x\\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0\\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x\\ y \end{bmatrix} + \begin{bmatrix} x_{i}\\ y_{i} \end{bmatrix}$$
(1)

With $i = 1, 2, \dots, 8$. Thus obtained

$$W = \bigcup_{i=1}^{8} W_i \tag{2}$$

Consider equation (2), an example of an Iterated Function System (IFS) generating Sierpinski Carpet is given below:

$$W_{1}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1/3 & 0\\0 & 1/3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix}$$
$$W_{2}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1/3 & 0\\0 & 1/3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}0\\1/3\end{bmatrix}$$

$$W_{3}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}0\\2\frac{1}{3}\end{bmatrix}$$
$$W_{4}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{1}{3}\\2\frac{1}{3}\end{bmatrix}$$
$$W_{5}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{2}{3}\\2\frac{1}{3}\end{bmatrix}$$
$$W_{6}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{2}{3}\\\frac{1}{3}\end{bmatrix}$$
$$W_{7}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{2}{3}\\\frac{1}{3}\end{bmatrix}$$
$$W_{7}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{2}{3}\\\frac{1}{3}\end{bmatrix}$$
$$W_{8}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{2}{3}\\0\end{bmatrix}$$
$$W_{8}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}\frac{1}{3}\\0\end{bmatrix}$$
$$W = W_{1} \cup W_{2} \cup W_{3} \cup W_{4} \cup W_{5} \cup W_{6} \cup W_{7} \cup W_{8}$$

In another reference, there is a discussion about the generation of the Sierpinski carpet. The Sierpinski carpet is a fractal constructed similarly to the Sierpinski triangle, but it uses squares instead of triangular objects [14].

The Sierpinski carpet can be constructed using a string with cell 1, and the iteration rule is as follows:

		0	0	0	1	1	1	
ł	$0 \rightarrow$	0	0	$0 , 1 \rightarrow$	1	0	1	ł
		0	0	0	1	1	1	

Because the Sierpinski carpet fractal model can be modified in various ways, this research aims to describe the construction of a 3D Sierpinski carpet model and utilize it for creating wallpaper patterns.

3. Research Methodology

In this study, the literature study method is employed by examining various sources such as books, journals, and internet resources that support this research, along with computational practices. The concept of geometric transformation is utilized to combine or vary patterns that have previously been generated from fractal geometry algorithms, resulting in a unified design of Siger and Lampung batik. The steps involved in designing wallpaper patterns using the Sierpinski3D fractal carpet are as follows:

- 1. Generating the Sierpinski 2D carpet pattern using commands and iterations in the Mathematica application.
- 2. Combining the generated fractal geometric patterns and transforming them into 3D.
- 3. Designing the integration of the created fractal geometric motifs to form a Wallpaper pattern.

4. Results and Discussion

This section discusses the results of the evaluation of the Sierpinski carpet from 2D to 3D and its transformation into a wallpaper for walls using the Sierpinski carpet concept by generating fractals into new patterns. This is achieved with the assistance of the Wolfram Mathematica® program. Before that, the pseudo code for constructing the 3D Sierpinski carpet fractal will be explained.

Pseudo code system							
m=ImageAdd[ImageResize[i,Scaled[0.9]],ImageResize[j,Scaled[0.86]]];							
n=3^(k=Floor[Log[3,Min[ImageDimensions[m]]]]);							
focus=ImageResize[ImageCrop[i,{n,n}]//ImageAdjust,n/3^#]&/@Range[k,0,-1];							
carpet1[focus_List,background_: Image[{{0}}]]:=Fold[ImageAssemble[{{#1,#1,#1},{#1,#2,#1},{#1,#1,#1}}]&,background,focus];							
fracimage=carpet1[focus];							
vtc={{0,0},{1,0},{1,1},{0,1}};							
$coords = \{\{\{0,0,0\},\{0,1,0\},\{1,1,0\},\{1,0,0\},\{\{0,0,0\},\{1,0,0\},\{1,0,1\},\{\{1,0,0\},\{1,1,0\},\{1,1,1\},\{1,0,1\},\{1,1,0\},\{1,1,1\},\{1,0,1\},\{1,1,1\},\{1,1,1\},\{1,0,1\},\{1,1,1,1\},\{1,1,1,1\},\{1,1,1,1\},\{1,1,1,1\},\{1,1,1\},\{1,1,1\},\{1,1,1\},\{1,1,1\}$							
Graphics3D[{Texture[fracimage],Polygon[coords,VertexTextureCoordinates2Table[vtc,{6}]]},Boxed2F							

alse]

After understanding the Pseudo code of the system, a programming flow will be provided using a flow chart diagram. A flow chart diagram represents the programming flow so that it can be understood in more detail. Below is the flow chart diagram for the Sierpinski 3D Program.



Figure 4. Flowchart for the 3D Sierpinski Program

4.1 Construction of 3D Sierpinski Carpet

The following is The 3D Sierpinski carpet fractal construction generated using the Wolfram Mathematica® application program and then assembled into a wallpaper form.

4.1.1 3D Sierpinski Carpet

The 3D Sierpinski carpet is created by extending the 2D Sierpinski carpet into three dimensions. This is achieved by introducing a third variable and dividing each side of the cube into nine smaller, congruent squares. The center square is removed, and the process is repeated for the remaining eight squares, which are further divided in the same way. Geometric transformations such as reflections, translations, and dilations create a fractal structure that resembles the Sierpinski cube.



Figure 5. Plain 3D Sierpinski Carpet

This set can be expressed as a combination of eight congruent subsets with the original set and has a scale factor of $\frac{1}{3}$. Thus, this subset has a value of k = 24 and a scale factor of $s = \frac{1}{3}$. It can be observed that the pattern of squares in this set will continue to repeat with progressively smaller scale factor values. By illustrating

$$W_{i} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} x_{i} \\ y_{i} \\ z_{i} \end{bmatrix}$$
(3)

With $i = 1, 2, \dots, 24$. Thus obtained

$$W = \bigcup_{i=1}^{24} W_i$$

Example of Iterated Function System (IFS) generating Sierpinski Carpet:

$$W_{1} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$W_{2} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1/3 \\ 1/3 \end{bmatrix}$$
$$W_{3} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 2/3 \\ 2/3 \end{bmatrix}$$

$$\begin{split} W_{4} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 1/3 \\ 0 \end{bmatrix} \\ W_{5} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 2/3 \end{bmatrix} \\ W_{6} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{7} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{10} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{11} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{12} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{12} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix} \\ W_{13} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 2/3 \\ 0 \end{bmatrix} \\ W_{14} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{14} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{15} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{15} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{15} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\ W_{16} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \\$$

$$\begin{split} W_{16} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 2/3 \\ 0 \end{bmatrix} \\ W_{17} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \\ 1/3 \end{bmatrix} \\ W_{18} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} \\ W_{19} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{20} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ W_{21} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 1/3 \\ 0 \end{bmatrix} \\ W_{22} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \\ 0 \end{bmatrix} \\ W_{23} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \\ 0 \end{bmatrix} \\ W_{24} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 1/3 \end{bmatrix} \\ W_{24} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \\ 1/3 \end{bmatrix} \\ \end{split}$$

4.1.2 3D Carpet Lena Pattern

The first pattern formed using the Lena image (see [16] for image) with Mathematica function (see [14] for detail).

carpet1[focus_List, background_:Image[{{0}}]]:= Fold[ImageAssemble[{{#1,#1,#1}, {#1,#2,#1}, {#1,#1,#1}}]&, background, focus];



Figure 6. 3D Carpet Lena Pattern

Figure 6 shows the Sierpinski carpet image with the base pattern of Lena formed on a clearly visible cube. The scale factor used is s = 1 forming the basic Sierpinski carpetmotif. The fractal symmetry of the Sierpinski carpet will be used to generate other motifs in three-dimensional form.

4.1.3 Carpet Sierpinski 3D Combination Pattern 1

The next motif uses a combination of two images, namely the Lena image and the Butterfly image (see [14] for image). The combined result of the two images is then formed into a Sierpinski carpet model using Mathematica functions.

carpet1[focus_List, background_:Image[$\{\{0\}\}$]]

:=Fold[ImageAssemble[{ $\{\#1,\#1,\#1\}, \{\#1,\#2,\#1\}, \{\#1,\#1,\#1\}\}$]&, background, focus].

Next, the model is given color functions.

Hue ColorFunction \rightarrow Hue



Figure 7. Carpet Sierpinski 3D Combination Pattern 1

4.1.4 Carpet Sierpinski 3D Combination Pattern 2

The second combination motif uses the Sierpinski carpet as a base but constructs a different motif. The following is the Mathematica function used:

carpet2[focus_List, background_:Image[$\{\{0\}\}$]]

:= Fold[ImageAssemble[{{#1, #1, #1}, {#1, #2, #1}, {#2, #2, #2}}]&, background, focus];

Next, the motif is evoked with the dominance of white and purple colors using function *Mathematica* ColorFunction \rightarrow (Blend[{White, Purple}]. The obtained results are as shown in the picture below:



Figure 8. Carpet Sierpinski 3D Combination Pattern 2

4.1.5 Carpet Sierpinski 3D Combination Pattern 3

The next motif uses the Sierpinski motif as its basis, and then the third combination model is formed using Mathematica functions.

carpet3[focus_List, background_:Image[$\{\{0\}\}$]]

 $:= Fold[ImageAssemble[\{ \#2, \#1, \#2\}, \{ \#1, \#2, \#1\}, \{ \#1, \#1, \#1\} \}] \&, background, focus];$

Then the model is given 'Rainbow' coloring using the Mathematica function:

ColorFunction \rightarrow "Rainbow", ColorRules $\rightarrow \{0 \rightarrow \text{White}\}$ Thus, the obtained motif is as shown in the picture below:



Figure 9. Carpet Sierpinski 3D Combination Pattern 3

4.1.6 Carpet Sierpinski 3D Combination Pattern 4

The next motif uses the Sierpinski motif as its basis, and then the third combination model is formed using Mathematica functions. carpet4[focus_List, background_:Image[$\{\{0\}\}$]

 $:= Fold[ImageAssemble[\{ \{ \#2, \#1, \#2 \}, \{ \#2, \#1, \#2 \}, \{ \#1, \#1, \#1 \} \}] \&, background, focus];$

Thus, the obtained motif is as shown in the picture below:



Figure 10. Carpet Sierpinski 3D Combination Pattern 4

5. Conclusion

The previous section indicates that the Sierpinski carpet fractal can be formed through mappings that exhibit self-similarity properties and can be modified into various batik or wallpaper motifs featuring geometric patterns. These motifs are obtained by manipulating mappings using the Mathematica® programming package. Additionally, tools within Mathematica, such as [ColorFunction], can be employed to color geometric patterns or motifs. In order to obtain geometric patterns used as motifs in wallpapers, this research has developed algorithms and programming using the Mathematica programming language. The generated program enables the discovery of a diverse range of derivative motifs.

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